effects on the transient airloading in the vicinity of the stagnation point of a body already in high-speed flow and then subjected to a blast-type aerodynamic disturbance could be investigated.

References

[1] Lemcke, B., "An investigation of the stagnation conditions in the shock-compression heater of a gun tunnel," Aeronaut. Res. Inst. Sweden Rept. 90 (1962).

[2] Slawsky, Z. J. and Seigel, A. E., "A two-stage driver for shock tubes and shock tunnels (*U*)," U.S. Naval Ordnance Test Station, NAVORD Rept. 5669 (1960).

An Erroneous Concept Concerning Nonlinear Aerodynamic Damping

Charles H. Murphy*
Ballistic Research Laboratories,
Aberdeen Proving Ground, Md.

In recent years, considerable interest has developed in the aerodynamic damping of symmetric configurations particularly those suitable for re-entry vehicles. Refined rigs have been developed in a number of this country's wind tunnels to measure this quantity for a wide variety of shapes. In most of these experiments, a model is forced to oscillate in a plane about a pivot support in a wind tunnel test section and the energy required to maintain this planar motion is measured and used to obtain values of $C_{M_q} + C_{M_{\dot\alpha}}$. Amplitude dependence can be determined either by forcing oscillations of different amplitudes about zero angle or forcing small-amplitude oscillations about a series of different trim angles.

In the course of this work, it has been noted that, for a large class of shapes at certain Mach numbers, the damping in pitch coefficients changes from destabilizing (positive values) at small amplitude to stabilizing (negative values) at large amplitudes. These data lead to the prediction that a freely oscillating missile should perform a limit oscillation with a fixed maximum amplitude. This is certainly true for a missile constrained to oscillate in a plane; indeed, these oscillations have been observed in tunnel tests. From this, the intuitively "obvious" statement has been made that missiles with this type of planar damping characteristics which are free to perform combined pitching and yawing motion will exhibit a limit cycle motion with a fixed maximum amplitude. It is the purpose of this note to show the fallacy of this assumption by use of a simple counter example.

The aerodynamic moment usually is assumed to be a function of the angles of attack and side slip α, β, the pitching and yawing angular velocities q, r, and the derivatives of these four quantities. For transient oscillations about the flight path, the angular velocities can be eliminated in the moment expansion by the approximations $\dot\alpha = q$, $\dot\beta = -r$. It can be shown that the assumption of rotational symmetry requires that the aerodynamic coefficients are functions of $\delta^2 = \alpha^2 + \beta^2$, $(\delta^2)^\bullet$, and $(\dot\alpha)^2 + (\dot\beta)^2$ and that of thesix possible cubic terms in the moment expansion two have a strong effect on the damping of the motion.[1] These two nonlinear terms will be used in our counter-example†; therefore,

$$C_m + iC_n = -i\left\{C_{M_\alpha}\xi + [C_{M_{q_0}} + C_{M_{\dot\alpha_0}} + a\delta^2]\xi' + b(\delta^2)'\xi\right\} \quad (1)$$

Received February 28, 1963.

* Aeronautical Research Engineer.

† The cubic term $\xi^2\bar\xi'$ of Ref. 1 can be converted to the form of Eq. (1) by the identity $(\delta^2)'\xi = (\xi\bar\xi)'\xi = \xi^2\bar\xi' + \delta^2\xi'$.

where

$$\xi = \beta + i\alpha \qquad \xi' = \dot\xi(l/V) \qquad (\delta^2)' = (\delta^2)^\bullet(l/V)$$

In Eq. (1), the first cubic damping moment term induces rotation about an axis normal to the derivative of the complex angle of attack while the second cubic term induces rotation about an axis normal to the complex angle of attack. For planar motion, $\xi = \delta e^{i\theta_0}$ and the complex angle of attack is directed along the same line as its derivative. Therefore,

$$C_m + iC_n = -ie^{i\theta_0}\left\{C_{M_\alpha}\delta + [C_{M_{q_0}} + C_{M_{\dot\alpha_0}} + (a+2b)\delta^2]\delta'\right\} \quad (2)$$

Note that wind tunnel tests would measure the combination $a + 2b$. For circular motion, however, $\xi = \delta_0 e^{i\theta}$ and the complex angle of attack is normal to its derivative.

$$C_m + iC_n = -i\delta_0 e^{i\theta}\left\{C_{M_\alpha} + i\theta'[C_{M_{q_0}} + C_{M_{\dot\alpha_0}} + a\delta_0^2]\right\} \quad (3)$$

The differential equation for ξ for constant drag, linear normal force, and the moment defined by Eq. (1) is

$$\xi'' + H_0[(1 + A\delta^2)\xi' + (\tfrac{1}{2})B(\delta^2)'\xi] - M\xi = 0 \quad (4)$$

where

$$H_0 = \rho Sl/2m[C_{N_\alpha} - 2C_D - k_t^{-2}(C_{M_{q_0}} + C_{M_{\dot\alpha_0}})]$$
$$A = -(\rho Sl/2m)k_t^{-2}H_0^{-1}a$$
$$B = -2(\rho Sl/2m)k_t^{-2}H_0^{-1}b$$
$$M = (\rho Sl/2m)k_t^{-2}C_{M_\alpha}$$

Under the quasi-linear assumptions,[2] the actual nonlinear angular motion is approximated by a pair of rotating, exponentially damped two-dimensional vectors whose damping exponents are functions of amplitude. Therefore,

$$\xi = K_1 e^{i\phi_1} + K_2 e^{i\phi_2} \quad (5)$$

where

$\phi_j = \phi_{j0} \pm (-M)^{1/2}s$
$K_j' = K_j\lambda_j(K_1^2, K_2^2)$
s = dimensionless arclength ($ds/dt = v/l$)

The general behavior of the motion can be obtained by the study of the $K_1^2 - K_2^2$ plane associated with Eq. (4) (the amplitude plane):

$$\frac{dK_2^2}{dK_1^2} = \frac{K_2^2\lambda_2}{K_1^2\lambda_1} = \frac{K_2^2[1 + BK_1^2 + AK_2^2]}{K_1^2[1 + AK_1^2 + BK_2^2]} \quad (6)$$

The motion described by Eq. (5) is a damped elliptical motion. Points on the coordinate axes of the amplitude plane ($K_1^2 = 0$, or $K_2^2 = 0$) represent circular motion and points on the line $K_1^2 = K_2^2$ correspond to planar motion. When A is negative, circular singularities exist at $(-A^{-1}, 0)$ and $(0, -A^{-1})$; when $A + B$ is negative, a planar singularity exists at $[-(A+B)^{-1}, -(A+B)^{-1}]$. The usual tests for the nature of a singularity[3] show that the planar singularity is a saddle point when

$$-1 < -A/B < 1 \quad (7)$$

and is a node otherwise.

For the lower half of this interval $(-1 < -A/B < 0)$, a circular singularity also exists and is a node. This interval on $-A/B$ was used in Ref. 1 in order to illustrate the existence of circular limit motions. If $-A/B$ is in the upper half of this interval, circular and planar singularities cannot exist at the same time. Figure 1 illustrates this possibility for $A + B$ negative.‡

‡ In Ref. 4, Eq. (1) is extended to include a cubic static moment ($C_{M_\alpha} = c_1 + c_2\delta^2$).

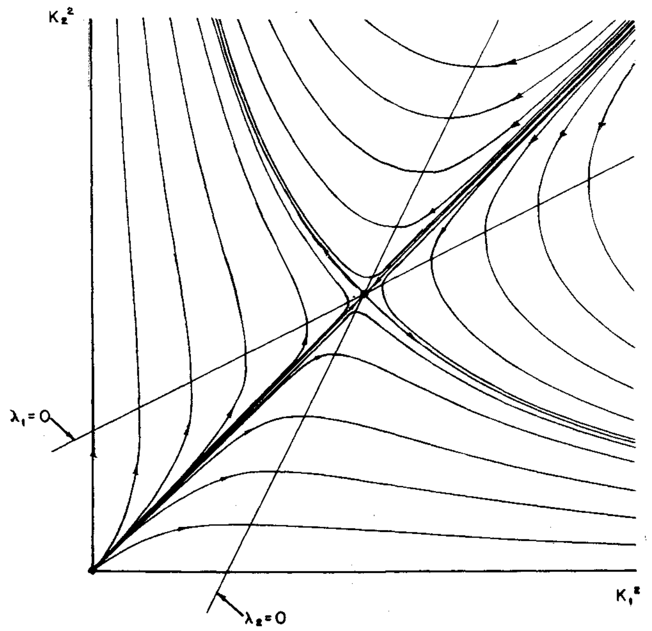

Fig. 1 Amplitude plane for unstable planar limit motion

According to the figure, exactly planar motions will go to a limit planar motion of maximum amplitude

$$K_1 + K_2 = 2|A + B|^{-1/2}$$

Large, nearly planar motion will decrease in amplitude, become elliptical and grow exponentially. Small motion and large, nearly circular motion immediately grow exponentially. Thus, the damping moment described by Eq. (1) would induce a planar limit motion in wind tunnel tests and would cause the angular motion to grow without bound in free flight. This example indicates the desirability of wind tunnel damping in pitch tests which allow the missile to move in a circular or elliptical motion.

References

[1] Murphy, C. H., "Circular pitching and yawing motion of cone configurations," *Proceedings of the Fourth AFBMD/STL Symposium* (Pergamon Press, New York, 1961), Vol. 2, pp. 328–336; also Ballistic Research Lab. Rept. 1071 (March 1959); also J. Aerospace Sci. **26,** 600–601 (1959).

[2] Murphy, C. H., "The prediction of nonlinear pitching and yawing motion of symmetric missiles," J. Aeronaut. Sci. **24,** 473–479 (1957); also Ballistic Research Lab. Rept. 995 (October 1956).

[3] Stoker, J. J., *Nonlinear Vibrations* (Interscience Publishers, Inc., New York, 1950).

[4] Murphy, C. H. and Hodes, B. A., "Planar limit motion of nonspinning symmetric missiles acted on by cubic aerodynamic moments," Ballistic Research Lab. Memo. Rept. 1358 (June 1961).

Results of Solid Rocket Motor Extinguishing Experiments

Richard W. Slocum Jr.*
American Space Industries, Inc., Palos Verdes, Calif.

An experiment was carried out to determine the feasibility of extinguishing a solid propellant rocket motor prior to normal burnout by injecting a suitable extinguishing chemical onto the burning surface of the grain. There, combustion would be stopped by either one or both of the following processes: 1) the temperature of the burning surface is lowered below the combustion temperature, and 2) the extinguishant acts as a chemical inhibitor of the combustion reactions.

Regarding extinguishants, a number of suggestions have been made, ranging all the way from water to more sophisticated chemicals that have been found effective in extinguishing more conventional types of chemical fires; however, few, if any, of these extinguishants have been tried in an operating motor. Specific mention may be made of certain halogenated hydrocarbons including some of the Freons,† e.g., FE 1301—bromotrifluoromethane ($CBrF_3$) and 114 B 2—dibromotetrafluoroethane ($CBrF_2$-$CBrF_2$), and bromochloromethane (CH_2BrCl). These chemicals are known to be excellent extinguishants of chemical fires.[1] It should be pointed out, however, that it is just as important for a "common use" extinguishant to have low toxicity as it is for it to have high extinguishing effectiveness. Thus there almost certainly are other compounds with greater extinguishing effectiveness than these, but which have not been classed as extinguishants because of their toxicity.

In the present case, bromochloromethane was selected as the extinguishant to be tried in an operating motor. This compound has a specific gravity of 1.95 and a boiling point of 152.6°F. It will be noted that this compound has one bromine atom per molecule. On the basis of data relating to flame speed in methane flames,[2] suggestions have been made that compounds with more than one bromine atom per molecule, or a greater proportion of bromine atoms, would be more effective as extinguishants. However, this is not supported by the data of Ref. 1 which show that on the average (of 7 compounds) those with 2 bromine atoms per molecule are not as effective as those with one, and none of those with 2 are as effective as bromochloromethane.

The motor used in the experiments had the following characteristics (when operating without injection of an extinguishant):

Propellant: 18% aluminum, 64% ammonium perchlorate, 18% PBAA and additives; flame temperature—5370°F; cylindrically shaped—4.75-in.-o.d. × 3.0-in.-i.d. × 7.38-in. long; flow rate—1.38 lb/sec

Chamber pressure: 430 psi

Thrust: 320 lb

Burning time: 3.7 sec

Injection was through the head end. To be sure the injectant reached the entire burning surface, a tube concentric with the longitudinal axis of the motor was used as injector. The injector extended the entire length of the grain and had an i.d. of 0.364 in. and o.d. of 0.540 in. 120-$\frac{1}{32}$-in. holes were drilled in the tube wall and spaced as follows: 30 circumferential rings of holes, each ring containing 4 holes (equally spaced around the circumference) and being perpendicular to the longitudinal axis of the injector; the relative phase of adjacent rings was 45°. Injection was initiated on command soon after motor ignition so that in use the injector was not damaged and could be reused.

Tests were carried out with two types of injector feed systems resulting in two different average flow rates of injectant *during motor operation.* These flow rates were 2.25 lb/sec and 6.15 lb/sec (±5%). In both cases, injectant tank pressure was 2000 psi (±10%). Calculations yield the following data: 1) average propellant velocity off the burning surface is 14.7 fps, 2) average injectant velocity at the surface of the injector is 26 fps at low flow and 70 fps at high.

When injection at the lower flow rate was initiated during motor operation and *continued until burnout,* the characteristic yellow-white exhaust plume remained basically yellow-white but trailed black "smoke"; this smoke is due to the

Received February 25, 1963.

* Physicist; presently Section Head, Propulsion Department, Systems Research and Planning Division, Aerospace Corporation, El Segundo, Calif.

† Dupont's trademark for its fluorocarbon compounds.